Fret Spacing Challenge

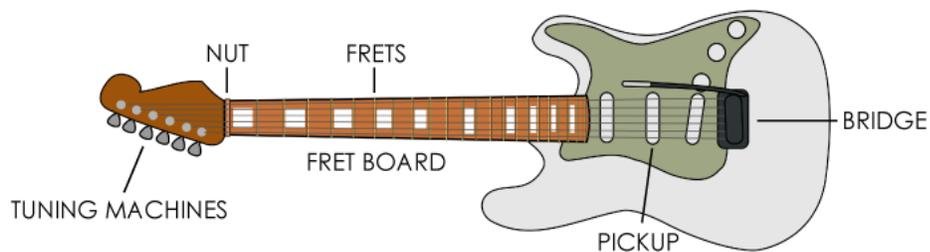Below is a diagram of an electric guitar based on a Fender Stratocaster.

Figure 1: Electric Guitar Parts

The frets on a guitar are the metal wires that rest on the fretboard. These can be seen in the photo of a guitar in Joe Garofalo's office.

Questions to Consider:

- What do you notice about the positions of the frets on this guitar?

- Why do you think they were placed in these positions?

- How are these fret positions determined?

The Challenge:

The fret spacing challenge is for you to answer the last question:

"How are these fret positions determined?"

Some Important Background Information:

There are four necessary pieces of information needed to answer *The Challenge*:

1. The western chromatic scale of notes contains 12 semitones, with notes referred to by the letters A through G, with some sharp (♯) and flat (b) notes added. A scale can start on any note, so far example, the notes A, Bb, B, C, C♯, D, Eb, E, F, F♯, G, Ab, and A form a chromatic scale consisting of one octave, starting and ending with an A note. Some notes can be referred to in two ways; for example, the note between C and D can be referred to either as C♯ or Db. All of these notes can be played on one string of a guitar by pressing down on the string and plucking it after each successive fret.

2. The two A notes in the above scale are an octave apart, with the first A note being lower in pitch than the last A note. The length of string vibrating when the lower A is played is twice the length of the string vibrating when the higher A is played. Pressing the string down on a fret essentially shortens the vibrating string length. So, the full length of string between the nut and the bridge of a guitar is twice that of the length of vibrating string note played by plucking the string while pressing down on the twelfth fret.

3. Frets should be placed on the fretboard so that the relative change in pitch between successive notes played by fretting the guitar is the same for any two successive notes. For example, the relative change in pitch between B and C should be the same as that between E and F.

4. The frequency of the note is proportional to the length of the string.

Simplifying the Challenge

There are several ways to determine where to place the frets so that all of the notes placed satisfy the equal change in pitch criterion, often referred to as equal temperament.

One way to simplify *The Challenge* is to break it into sub tasks: (1) determine fret spacing for a two-fret instrument, (2) determine fret spacing for a three-fret instrument, and (3) create an algorithm for fret spacing for a regular guitar.

Task 1: Two-Fret Instrument.

The two-fret instrument is to include one fret midway between the nut at the head of the guitar and bridge and another fret placed between the nut at the head of the guitar and the midpoint fret. Each string would then be able to produce three notes – one when the string was played open, a note an octave higher when the string was fretted at the center, and a third note when the instrument was fretted at the second fret to be placed. Remember that the change in pitch from the open note to the non-octave note should be the same as the change in pitch from the new note to the octave note.

With a two-fret instrument, there are three notes: Note 1, Note 2, and Note 3. A two-fret instrument therefore has the ability to play three notes: the open string which produces Note 1, the first fret (the one closest to the nut at the head of the guitar) creates Note 2, and the

second fret (the one in the middle of the fret board) produces Note 3. The ratio of the frequency of Note 1 to the frequency of Note 2 should be same as the ratio of the frequency of Note 2 to the frequency of Note 3.

Task 2: The Three-fret Instrument.

This task builds on the two-fret task. The 3-fret instrument is to include one fret midway between the nut and bridge and two other frets between the nut at the head of the guitar and the midpoint fret, so that the string could produce four notes satisfying the equal change in pitch criterion.

Task 3: Create an algorithm.

This task asks you to create an algorithm for placing frets on a regular guitar that will allow you to play all 12 semitones on a string.